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NONLINEAR TRIPLE-WAVE INTERACTIONS IN SATURATED POROUS MEDIA

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Present-day geophysical problems involving both analysis of the interaction between geophysical fields [1] and technological matters concerning the seismoacoustic effect on hydrocarbon deposits [2, 3] as well as the development of a synergetic concept of a geological medium [4] necessitate theoretical studies of the mechanisms of nonlinear wave propagation and interaction in porous media. The commonly accepted methodology of nonlinear wave theory [5] assumes the transformation of the equations of mechanics conservation laws corresponding to a given model into nonlinear evolution equations, which appear as conditions of asymptotic solvability and describe the propagation of envelope waves over long time periods.

The goal of papers [6–9] was to derive nonlinear evolution equations for various models of porous media and to analyze the resonance effects described by the solutions of these equations. In [10, 11] a generalization of the classical Fraenkel–Biot–Nikolaevskii model [12, 13], which takes into account the dispersion factor (viscous shear stresses in the liquid phase), was proposed and a mathematical foundation of the asymptotic transformation to nonlinear evolution equations was given for constructing one-phase solutions.

The present paper constructs a many-phase asymptotic solution determined by a series of elastic waves and considers a triple-wave resonance effect. Solution of this problem with allowance for to the dispersion properties of the medium in a weak-nonlinearity approximation assumes determination of the conditions for resonance interaction and analysis of the mechanisms of energy redistribution in the system of resonance triads [14, 15]. It is demonstrated that modulated-wave propagation can be described by the Korteweg–de Vries–Burgers equation. For resonance triads, the Maenly–Rough law of energy conservation holds.

1. Definition of Porous-Medium Model. Consider a viscoelastically deformable porous medium consisting of an elastic frame, a surface-bound frame of viscous liquid, and a viscous fluid (incompressible liquid or ideal gas) phase.

Note that in this case, unlike the conventional consideration of a pore fluid with $P_{ij} = -p\delta_{ij}$, the shear stresses in the fluid are taken into account:

$$P_{ij} = -p\delta_{ij} + \nu_f\Big[\frac{\partial v_{fi}}{\partial x_j} + \frac{\partial v_{fj}}{\partial x_i} - \frac{2}{3}\frac{\partial v_{fk}}{\partial x_k}\delta_{ij}\Big]$$

[P_{ij} is the tension tensor in the fluid phase; p is the pressure; v is the rate vector; ν is the viscosity; index f can take values of l (liquid) or g (gas), corresponding to porous medium saturation by an incompressible liquid or an ideal gas].

The porous-medium frame and bound liquid form an effective viscoelastic solid phase that displays the elastic characteristics of the frame and the viscous characteristics of the liquid. In this case, the frame and bound liquid have the same velocity, temperature, and pressure. Allowing for the viscoelastic properties we derive a rheological relation for the solid phase:

$$\sigma_{ij} = Ke_{kk}\delta_{ij} + 2G(e_{ij} - e_{kk}\delta_{ij}/3) + \beta_s Kp\delta_{ij} - \varphi_s KT_s\delta_{ij} + \alpha m\nu_\alpha\Big[\frac{\partial v_{si}}{\partial x_j} + \frac{\partial v_{sj}}{\partial x_i} - \frac{2}{3}\frac{\partial v_{sk}}{\partial x_k}\delta_{ij}\Big],$$

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where σ_{ij} is the tensor of effective stresses; e_{ij} is the deformation tensor; K is the module of volumetric elasticity; G is the shear modulus; β is the compressibility coefficient; φ is the expansion coefficient; m is the porosity; α is the volumetric fraction of the bound liquid; T is the temperature; subscript s is the solid phase and α is the bound liquid.

Let us now formulate a system of determining equations in dimensionless form. Let us introduce the dimensionless variables and the parameters:

$$x' = x/x_0, \quad t' = t/t_0, \quad u' = u/x_0, \quad \rho' = \rho/\rho_0, \quad P' = P/K_0, \quad \sigma' = \sigma/K_0, \quad T' = T/\theta_0,$$

$$\beta' = \beta K_0, \quad \varphi' = \varphi\theta_0, \quad K' = K/K_0, \quad G' = G/K_0, \quad v' = v/v_0, \quad E' = E/v_0^2, \quad \nu' = \nu/(K_0 t_0),$$

$$\lambda' = \lambda\theta_0 t_0/(x_0^2 v_0^2 \rho_0), \quad C' = C\theta_0/v_0^2, \quad R' = R\theta_0/v_0^2, \quad \chi' = \chi\theta_0 t_0/(v_0^2\rho_0), \quad k' = kx_0, \quad \omega' = \omega t_0$$

$$[x_0 = v_0 t_0, \quad t_0 = \rho_0 æ/\nu_f, \quad v_0 = (K_0/\rho_0)^{1/2}].$$

Here æ is the permeability; ρ is the density; u is the shear vector; E is the internal energy; λ is the heat conductivity coefficient; C is the heat capacity; R is the universal gas constant; χ is the interphase heat exchange coefficient; k is the wave vector; ω is the frequency.

The values of the dimensionless parameters can be estimated using characteristic values of rock constants [13]:

$$K_0 \sim 10^8\text{–}10^9 \text{ Pa}, \quad \theta_0 \sim 10^2\text{–}10^3 \text{ K}, \quad \rho_0 \sim 10^3 \text{ kg/m}^3, \beta \sim 10^{-10}\text{–}10^{-9} \text{ Pa}^{-1}, \quad \varphi \sim 10^{-6}\text{–}10^{-3} \text{ K}^{-1},$$

$$C \sim 10^3 \text{ J/(kg}\cdot\text{K)}, \quad \lambda \sim 10^0 \text{ W/(m}\cdot\text{K)}, \quad \nu \sim 10^{-5}\text{–}10^{-3} \text{ Pa}\cdot\text{sec}, \quad æ \sim 10^{-15}\text{–}10^{-12} \text{ m}^2.$$

For these parameters we have

$$t_0 \sim 10^{-9}\text{–}10^{-6} \text{ sec}, \quad v_0 \sim 10^3 \text{ m/sec}, \quad x_0 \sim 10^{-6}\text{–}10^{-3} \text{ m}, \quad \nu' \sim 10^{-8}\text{–}10^{-2},$$

$$\beta' \sim 10^{-2}\text{–}10^0, \quad \varphi' \sim 10^{-3}\text{–}10^0, \quad C' \sim 10^0, \quad \lambda' \sim 10^{-7}\text{–}10^{-4}.$$

The small dimensionless parameter obtained by analyzing the dimensions

$$\varepsilon = (\nu_f')^{1/2} = (\nu_f/(K_0 t_0))^{1/2} \equiv \nu_f/(K_0\rho_0 æ)^{1/2}$$

is a combination of the average fluid viscosity, elasticity modulus, density, and medium permeability and determines the scale of the manifestation of the dispersion properties.

Below, the primes will be omitted. Using the above estimates of dimensionless parameters we assume $\nu = \varepsilon^2\tilde{\nu}$ and $\lambda = \varepsilon^2\tilde{\lambda}$. Omitting the index $\sim$, we write a set of equations of mass, energy, and momentum conservation $(i, j = 1, 2, 3)$:

$$\partial((1-\alpha)m\rho_f)/\partial t + \nabla_x((1-\alpha)m\rho_f v_f) = 0,$$

$$\partial(\alpha m\rho_\alpha + (1-m)\rho_s)/\partial t + \nabla_x((\alpha m\rho_\alpha + (1-m)\rho_s)v_s) = 0,$$

$$(1-\alpha)m\rho_f[\partial/\partial t + < v_f, \nabla_x >]v_{fi} - (1-\alpha)m\partial P_{ij}/\partial x_j + m^2(1-\alpha)^2(v_{fi} - v_{si}) = 0,$$

$$(\alpha m\rho_\alpha + (1-m)\rho_s)[\partial/\partial t + < v_s, \nabla_x >]v_{si} - \partial\sigma_{ij}/\partial x_j$$

$$-(1-(1-\alpha)m)\partial P_{ij}/\partial x_j - m^2(1-\alpha)^2(v_{fi} - v_{si}) = 0, \quad \partial u_i/\partial t - v_{si} = 0,$$

$$\partial\sigma_{ij}/\partial t = K\delta_{ij}\partial e_{kk}/\partial t + 2G\partial(e_{ij} - e_{kk}\delta_{ij}/3)/\partial t + \beta_s\delta_{ij}K\partial p/\partial t \tag{1.1}$$

$$-\varphi_s\delta_{ij}K\partial T_s/\partial t + \varepsilon^2\alpha m\nu_\alpha\partial[\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i - (2/3)(\partial v_{sk}/\partial x_k)\delta_{ij}]/\partial t,$$

$$\partial e_{ij}/\partial t - (\partial v_{si}/\partial x_j + \partial v_{sj}/\partial x_i)/2 = 0,$$

$$m(1-\alpha)\rho_f[\partial/\partial t + < v_f, \nabla_x >]E_f = m(1-\alpha)P_{ij}\partial v_{fi}/\partial x_j$$

$$+m^2(1-\alpha)^2 \mid v_f - v_s \mid^2 - \chi(T_f - T_s) + \varepsilon^2\nabla_x(m(1-\alpha)\lambda_f\nabla_x)T_f,$$

$$(1-m)\rho_s[\partial/\partial t + < v_s, \nabla_x >]E_s + \alpha m\rho_\alpha[\partial/\partial t + < v_s, \nabla_x >]E_\alpha$$

$$= [\sigma_{ij} + (1-(1-\alpha)m)P_{ij}]\partial v_{si}/\partial x_j + \chi(T_f - T_s) + \varepsilon^2\nabla_x((1-m)\lambda_s + \alpha m\lambda_\alpha)\nabla_x T_s.$$

The rheological and thermodynamic relations are of the form

$$\begin{gathered}
\rho_l = \rho_{l0}(1 + \beta_l(p - p_0) - \varphi_l(T_l - T_{l0})), \quad \rho_g = p/(RT_g), \\
\rho_\alpha = \rho_{l0}(1 - \beta_\alpha(\sigma^s_{kk}/3 - \sigma_0) - \varphi_\alpha(T_s - T_{s0})), \quad v_f = (v_{f1}, v_{f2}, v_{f3}), \\
\rho_s = \rho_{s0}(1 - \beta_s(\sigma^s_{kk}/3 - \sigma_0) - \varphi_s(T_s - T_{s0})), \quad v_s = (v_{s1}, v_{s2}, v_{s3}), \\
P_{ij} = -p\delta_{ij} + \varepsilon^2 \nu_f[\partial v_{fi}/\partial x_j + \partial v_{fj}/\partial x_i - (2/3)(\partial v_{fk}/\partial x_k)\delta_{ij}], \\
\rho_l dE_l = \rho_l C_l dT_l + (p/\rho_l)d\rho_l - \varphi_l T_l dp, \quad E_g = C_g T_g, \\
\rho_s dE_s = \rho_s C_s dT_s + \sigma^s_{ij} de_{ij} + \varphi_s T_s d\sigma^s_{kk}/3, \quad \sigma^s_{ij} = \sigma_{ij}/(1 - (1 - \alpha)m) + P_{ij}, \\
\rho_\alpha dE_\alpha = \rho_\alpha C_\alpha dT_s - \sigma^s_{kk}/(3\rho_\alpha)d\rho_\alpha + \varphi_\alpha T_s d\sigma^s_{kk}/3
\end{gathered} \tag{1.2}$$

(σ^s_{ij} is the tensor of the true stresses in the solid phase).

System (1.1), (1.2) is a mainly hyperbolic and closed set of equations for the unknown tensor functions σ_{ij} and e_{ij}, vector functions u, v_s, and v_f, and scalar functions m, p, T_s, and T_f.

Further, we restrict ourselves to considering Cauchy's problem with the following initial data

$$\begin{gathered}
u_i \mid_{t=0} = u^0_i, \quad v_{si} \mid_{t=0} = v^0_{si}, \quad v_{fi} \mid_{t=0} = v^0_{fi}, \quad m \mid_{t=0} = m^0, \\
p \mid_{t=0} = p^0, \quad T_f \mid_{t=0} = T^0_f, \quad T_s \mid_{t=0} = T^0_s, \quad e_{ij} \mid_{t=0} = [\partial u^0_i/\partial x_j + \partial u^0_j/\partial x_i]/2, \\
\sigma_{ij} \mid_{t=0} = K e_{kk}\delta_{ij} \mid_{t=0} + 2G(e_{ij} - (1/3)e_{kk}\delta_{ij}) \mid_{t=0} + \beta_s K p^0 \delta_{ij} \\
-\varphi_s K T^0_s \delta_{ij} + \varepsilon^2 \alpha m^0 \nu_\alpha[\partial v^0_{si}/\partial x_j + \partial v^0_{sj}/\partial x_i - (2/3)(\partial v^0_{sk}/\partial x_k)\delta_{ij}].
\end{gathered} \tag{1.3}$$

2. Construction of a Many-Phase Asymptotic Solution.

Theorem 2.1. *The asymptotic solution for* mod $O(\varepsilon)$ *of Cauchy's problem for system* (1.1), (1.2) *with the initial data*

$$U \mid_{t=0} = U_b \mid_{t=0} + \varepsilon \sum_{i=1}^{N} H^i \overset{1}{U^i_0} (S_{i0}(x)/\varepsilon, x) \tag{2.1}$$

is of the form

$$U = U_b(x,t) + \varepsilon \sum_{i=1}^{N} H^i \overset{1}{U^i} (S_i/\varepsilon,\ x,\ t), \tag{2.2}$$

where U is the vector function of the values sought after:

$$U(\tau, x, t) = (m, v_{fi}, v_{si}, p, \sigma_{ij}, e_{ij}, T_f, T_s, u_i), \quad i, j = 1,\ 2,\ 3;$$

$U_b(x,t)$ is the slow background (the average of the U solution of the initial set of equations); $\overset{1}{U^i_0}(\tau_i, x)$ are scalar, real, C^∞, x-finite, 2π-periodic over $\tau_i = S_i/\varepsilon$ functions; the phases $S_i(x,t)$ are the solutions of Cauchy's problem for the Hamiltonian-Jacobi equation

$$\partial S_i/\partial t + \omega_i(x, t, \nabla_x S_i) = 0, \qquad S_i \mid_{t=0} = S_{i0}(x). \tag{2.3}$$

The ω_i frequencies obey the dispersion relation

$$\text{Det } A(S_{it}, S_{ix}, x, t) = 0; \tag{2.4}$$

$H^i(x,t)$ are zero-vectors corresponding to frequencies $\omega_i(x,t,k)$; $\overset{1}{U^i}(\tau_i, x, t)$ are any 2π-periodic in τ_i functions with a zero average; A is the symbol of the linearized operator $A(t, x, \partial/\partial t, \nabla_x)$ of the initial system of U_b background equations.

Note that the mainly hyperbolic system of equations and the form of the asymptotic solution chosen for this system predetermine the maximum number of the real roots of the dispersion equations.

The mod $O(\varepsilon^2)$ asymptotics is constructed on an assumption similar to the condition for the minor denominators of the Kolmogorov–Arnold–Mozer (KAM) theory [15, 16].

Condition A (KAM type). The positive constants c and μ are such that for all integers $n_j, j = 1, \ldots, N$ the following relation holds:

$$|\text{Det } A(t, x, -\omega, k)| \geqslant c\Big(\sum_{j=1}^{N} n_j^2\Big)^{-\mu}, \quad |(n_1, \ldots, n_N)| \neq 1.$$

In this case $k = \sum_{j=1}^{N} n_j k^{(j)}$; the nonzero vectors $k^{(j)} \in R^3$, $\omega = \sum_{j=1}^{N} n_j \omega_j(k^{(j)})$.

When this condition is satisfied, no wave resonance interaction occurs.

Theorem 2.2. *Let condition A be satisfied. Then the asymptotic solution for* mod $O(\varepsilon^2)$ *of Cauchy's problem* (1.1)–(2.1) *is of the form*

$$U = U_b(x,t) + \varepsilon \sum_{i=1}^{N} H^i \overset{1}{U^i} (\tau_i, x, t) + \varepsilon^2 (\overset{2}{U} (\tau, x, t) + Q(x, t, \varepsilon)),$$

where the vector $\tau = (\tau_1, \ldots, \tau_N)$. *The real scalar* 2π*-periodic* $\overset{1}{U^i} (\tau_i, x, t)$ *functions with* τ_i *zero averages are the solution of Cauchy's problem for the Korteweg–de Vries–Burgers equation*

$$d \overset{1}{U^i} /dt_A + a_1^i \partial^2 \overset{1}{U^i} /\partial\tau_i^2 + a_2^i \overset{1}{U^i} \partial \overset{1}{U^i} /\partial\tau_i + a_3^i \overset{1}{U^i} + \varepsilon\Big[a_4^i \partial^3 \overset{1}{U^i} /\partial\tau_i^3 + a_5^i \overset{1}{U^i} \partial^2 \overset{1}{U^i} /\partial\tau_i^2$$

$$+ a_6^i (\overset{1}{U^i})^2 \partial \overset{1}{U^i} /\partial\tau_i + a_7^i (\overset{1}{U^i})^2 + a_8^i (\partial \overset{1}{U^i} /\partial\tau_i)^2 + a_9^i \partial \overset{1}{U^i} /\partial\tau_i\Big] = 0, \qquad \overset{1}{U^i}|_{t=0} = \overset{1}{U_0^i}\,. \tag{2.5}$$

Here $\overset{2}{U} (\tau, x, t)$ *is a* C^∞ *function over the set of variables that is* 2π*-periodic with a* τ*-zero average; the* d/dt_A *operator is a total derivative along the characteristics satisfying Eq.* (2.3); Q *is the infinitely differentiable bounded function; the coefficients* a_j^i $(j = 1$–$9)$ *are determined from the construction.*

3. Resonance Triads. Equation (2.5) can be used to describe the propagation of modulated waves that do not take part in nonlinear interaction. However, a number of experimental facts are known [2, 3, 17] that attest to processes of new-wave generation due to the nonlinear interaction of elastic longitudinal and transverse waves in rocks. The problem of the theoretical study of this effect can be solved using conventional methods of nonlinear physics that consider schemes of triple-wave interactions [5, 14, 15, 18].

It is obvious that the following are necessary conditions for resonance wave interaction:

$$\omega^{(1)} + \omega^{(2)} = \omega^{(3)}, \qquad k^{(1)} + k^{(2)} = k^{(3)} \tag{3.1}$$

[$\omega^{(i)} = \omega^{(i)}(k^{(i)})$ and $k^{(i)}$ are the frequency and wave vector of the triplet of interacting waves, respectively].

A triple-wave resonance interaction in the model of a porous medium is described in terms of the generalized system of Korteweg–de Vries–Burgers equations:

$$\frac{d \overset{1}{U^i}}{dt_{Ai}} + a_1^i \frac{\partial^2 \overset{1}{U^i}}{\partial\tau^2} + a_2^i \overset{1}{U^i} \frac{\partial \overset{1}{U^i}}{\partial\tau} + a_3^i \overset{1}{U^i}$$

$$+ \varepsilon\left[a_4^i \frac{\partial^3 \overset{1}{U^i}}{\partial\tau^3} + a_5^i (\overset{1}{U^i})^2 + a_6^i \Big(\frac{\partial \overset{1}{U^i}}{\partial\tau}\Big)^2 + a_7^i \overset{1}{U^i} \frac{\partial^2 \overset{1}{U^i}}{\partial\tau^2} + a_8^i (\overset{1}{U^i})^2 \frac{\partial \overset{1}{U^i}}{\partial\tau} + a_9^i \frac{\partial \overset{1}{U^i}}{\partial\tau}\right] + W^i = 0, \tag{3.2}$$

where

$$W^1 = \gamma_1/(2\pi) \frac{\partial}{\partial\tau} \int_0^{2\pi} \overset{1}{U^2} (\xi, x, t) \overset{1}{U^3} (\tau + \xi, x, t) d\xi;$$

$$W^2 = \gamma_2/(2\pi) \frac{\partial}{\partial\tau} \int_0^{2\pi} \overset{1}{U^1} (\xi, x, t) \overset{1}{U^3} (\tau + \xi, x, t) d\xi;$$

$$W^3 = \gamma_3/(2\pi)\frac{\partial}{\partial\tau}\int\limits_0^{2\pi} \overset{1}{U}{}^1(\xi,x,t)\,\overset{1}{U}{}^2(\tau-\xi,x,t)\,d\xi.$$

In this case, the constants γ_i are determined from the construction and depend on the medium's parameters and the equilibrium background state, frequencies, and the wave vectors of the resonance triad.

For system of Eqs. (3.2) the laws of energy conservation (analogs of the Maenly–Rough relations [14, 15]) hold:

$$\frac{d}{dt_{Aj}}\mathcal{E}_j + \frac{\gamma_j}{\gamma_3}\frac{d}{dt_{A3}}\mathcal{E}_3 = 0, \qquad j = 1,2, \qquad \frac{d}{dt_{A1}}\mathcal{E}_1 + \frac{d}{dt_{A2}}\mathcal{E}_2 + (\gamma_1+\gamma_2)\Big/\gamma_3\frac{d}{dt_{A3}}\mathcal{E}_3 = 0, \tag{3.3}$$

where $\mathcal{E}_j(t)$ is the energy of a wave with phase S_j averaged over fast oscillations

$$\mathcal{E}_j = \frac{1}{2\pi}\Bigg\{\int\limits_0^{2\pi}\Big(\overset{1}{U}{}^j\Big)^2 d\tau + \int\limits_0^{t_{Aj}}\int\limits_0^{2\pi}\Big[\,|\,a_1^j\,| + \varepsilon(a_6^j - 2a_7^j)\,\overset{1}{U}{}^j\Big]\Big(\frac{\partial \overset{1}{U}{}^j}{\partial\tau}\Big)^2 d\tau\, dt_{Aj}$$
$$+ \int\limits_0^{t_{Aj}}\int\limits_0^{2\pi}[a_3^j + \varepsilon a_5^j\,\overset{1}{U}{}^j]\Big(\overset{1}{U}{}^j\Big)^2 d\tau\, dt_{Aj}\Bigg\}, \qquad j = 1,2,3.$$

Conservation laws (3.3) allow the wave interaction character to be analyzed from the standpoint of energy redistribution [15, 18] by examining, for simplicity, the x-independent Cauchy data $\overset{1}{U}{}^j|_{t=0} \equiv \overset{1}{U}{}^j_0(\tau)$. Then, due to the solution's uniqueness, $\overset{1}{U}{}^j(t,\tau)$ are independent of the slow variable x, i.e., $d\overset{1}{U}{}^j(t,\tau)/dt_{Aj} \equiv d\overset{1}{U}{}^j(t,\tau)/dt$, $j = 1, 2, 3$.

In this case the following variants are possible:

1. $\gamma_1,\gamma_2,\gamma_3 > 0$ or $\gamma_1,\gamma_2,\gamma_3 < 0$. Hence

$$\mathcal{E}_1(t) = \mathcal{E}_1(0) - \frac{\gamma_1}{\gamma_3}\mathcal{E}_3(t), \quad \mathcal{E}_2(t) = \mathcal{E}_2(0) - \frac{\gamma_2}{\gamma_3}\mathcal{E}_3(t), \quad \mathcal{E}_j(t) + \frac{\gamma_1+\gamma_2}{\gamma_3}\mathcal{E}_3(t) = \mathcal{E}_j(0), \quad j = 1,2.$$

Here a portion of the energy of two initial waves is "pumped" into the forming third wave. Since $(\gamma_1+\gamma_2)/\gamma_3 > 0$, the total system energy remains constant, so that the amplitudes of the initial waves decrease with increasing amplitudes of the resulting waves.

2. $\gamma_1 > 0, \gamma_2, \gamma_3 < 0$ or $\gamma_1 < 0, \gamma_2, \gamma_3 > 0$. Hence,

$$\mathcal{E}_1(t) = \mathcal{E}_1(0) + \frac{\gamma_1}{|\,\gamma_3\,|}\mathcal{E}_3(t), \quad \mathcal{E}_2(t) = \mathcal{E}_2(0) - \frac{|\,\gamma_2\,|}{|\,\gamma_3\,|}\mathcal{E}_3(t), \quad \mathcal{E}_j(t) + \frac{\gamma_1+\gamma_2}{\gamma_3}\mathcal{E}_3(t) = \mathcal{E}_j(0), \quad j = 1,2.$$

In this case, the forming third wave "takes" the energy portion (γ_2/γ_3) from the second one and "gives" the portion (γ_1/γ_3) to the first wave. The total energy of the system remains constant $[(\gamma_1+\gamma_2)/\gamma_3 > 0]$.

Similarly, with $\gamma_2 > 0$, $\gamma_1,\gamma_3 < 0$ or $\gamma_2 < 0$, $\gamma_1,\gamma_3 > 0$ the resulting wave "takes" the energy portion (γ_1/γ_3) from the first wave, and "gives" the portion (γ_2/γ_3) to the second one.

3. As noted above, the energy portion from the initial waves is "pumped" via the new wave to another initial wave. However, since the total system energy is not preserved $[(\gamma_1+\gamma_2)/\gamma_3 < 0]$, there is a potential for unlimited growth of the amplitude of interacting waves, which is sure to cause explosive instability in a finite time.

4. Triple-wave resonance fails. In this case, $\gamma_3 > 0$, $\gamma_1,\gamma_2 < 0$, or $\gamma_3 < 0, \gamma_1, \gamma_2 > 0$. Hence,

$$\mathcal{E}_1(t) = \mathcal{E}_1(0) + \frac{|\,\gamma_1\,|}{|\,\gamma_3\,|}\mathcal{E}_3(t), \quad \mathcal{E}_2(t) = \mathcal{E}_2(0) + \frac{|\,\gamma_2\,|}{|\,\gamma_3\,|}\mathcal{E}_3(t),$$

i.e., $\mathcal{E}_3(t) \equiv 0$, and in a first approximation triple-wave resonance is impossible.

Comment. Analyzing triple-wave interaction in terms of energy redistribution we formulated the necessary conditions for realizing various interaction regimes. The question of sufficient conditions calls for construction and study of the solution of Cauchy's problem for system (3.2), which is beyond the scope of our paper.

4. The Resonance Interaction of Longitudinal and Transverse Waves. In order to specify the conditions for resonance interaction (3.1) let us consider the frequencies of longitudinal and transverse waves. Performing dispersion analysis of the model, we deduce explicit expressions for the frequencies of longitudinal and transverse waves [10, 11]. We restrict ourselves to the case of zero background velocities of the solid and fluid phases ($v_{fi}^{(0)} = v_{si}^{(0)} = 0, \quad i = 1, 2, 3$).

Dispersion relation (2.4) is of the form

$$\text{Det } A(t, x, -\omega, k) = (1-\alpha)^5 m^4 \mathcal{P}_4 \mathcal{P}_s^2 \rho_l^2 \omega^{17} / (3(1-(1-\alpha)m))$$

for the liquid-saturated porous medium, and

$$\text{Det } A(t, x, -\omega, k) = (1-\alpha)^5 m^4 \mathcal{P}_4 \mathcal{P}_s^2 \rho_g^2 \omega^{17} / (3RT_g^2(1-(1-\alpha)m))$$

for the gas-saturated porous one.

The frequencies of the first and second types of longitudinal waves (forward and backward) are determined by a fourth-degree polynomial $\mathcal{P}_4$:

$$\mathcal{P}_4 = \Gamma_0\omega^4 - \Gamma_1\omega^2 + \Gamma_2.$$

In this case the coefficient Γ_0 is a constant that is independent of the wave vector and is determined by the equilibrium background and the medium's parameters. Coefficients Γ_1 and Γ_2 are written as

$$\Gamma_1 = c_1|k|^2 + æ_1\sigma_{ij}^{(0)}k_ik_j, \quad \Gamma_2 = |k|^2(c_2|k|^2 + æ_2\sigma_{ij}^{(0)}k_ik_j),$$

where k_i are the coordinates of the wave vector k; the constants $c_1, æ_1$ and $c_2, æ_2$ are also determined by the equilibrium background and the medium's parameters. Thus, the frequencies of the first and second types of longitudinal waves (forward and backward) take the form

$$\omega_1 = \pm((\Gamma_1 + (\Gamma_1^2 - 4\Gamma_0\Gamma_2)^{1/2})/(2\Gamma_0))^{1/2}, \quad \omega_2 = \pm((\Gamma_1 - (\Gamma_1^2 - 4\Gamma_0\Gamma_2)^{1/2})/(2\Gamma_0))^{1/2},$$

and the multiple frequencies of transverse waves (forward and backward) with different polarizations are:

$$\omega_3 = \omega_4 = \pm(G/\rho_{s\alpha})^{1/2}|k|,$$

where $\rho_{s\alpha} = \alpha m\rho_\alpha + (1-m)\rho_s$. Consider now the case of a resonance triad. Let $\omega^{(1)} = \omega_3$ be the transverse-wave frequency; $\omega^{(2)} = \omega_2$ the frequency of the second type of longitudinal wave, and $\omega^{(3)} = \omega_1$ the frequency of the first type of transverse wave. We determine the vectors. Let $\Lambda^{(j)}$ be eigenvectors of the matrix of the background stress state tensor, i.e.:

$$\sigma_{ij}^{(0)}\Lambda^{(j)} = \lambda^{(j)}\Lambda^{(j)}, \quad |\Lambda^{(j)}| = 1, \quad j = 1, 2, 3$$

($\lambda^{(j)}$ are eigenvalues). We assume that

$$k^{(1)} = n_1\Lambda^{(1)} \text{ и } k^{(2)} = n_2\Lambda^{(2)}$$

(n_1 and n_2 are integers). Hence, due to (3.1):

$$k^{(3)} = n_1\Lambda^{(1)} + n_2\Lambda^{(2)}.$$

In this case, condition (3.1) is of the form

$$\begin{aligned}(G/\rho_{s\alpha})^{1/2} &+ Z((c_1 + æ_1\lambda^{(2)} - ((c_1 + æ_1\lambda^{(2)})^2 - 4\Gamma_0(c_2 + æ_2\lambda^{(2)}))^{1/2})/(2\Gamma_0))^{1/2} \\ &= ((c_1(1+Z^2) + æ_1(\lambda^{(1)} + Z^2\lambda^{(2)}) + ((c_1(1+Z^2) + æ_1(\lambda^{(1)} + Z^2\lambda^{(2)}))^2 \\ &- 4\Gamma_0(1+Z^2)(c_2(1+Z^2) + æ_2(\lambda^{(1)} + Z^2\lambda^{(2)})))^{1/2})/(2\Gamma_0))^{1/2}, \end{aligned} \tag{4.1}$$

where $Z = n_2/n_1$.

TABLE 1

	I	II	⊥
I	II(1), ⊥(3)	II(2)	⊥(2)
II	I(1),⊥(2)	⊥(2)	⊥(4)
⊥	I(1)	II(2)	—

TABLE 2

	I	II	⊥
I	⊥ (2)	II(1)	⊥(4)
II	I(3), ⊥ (3)	⊥ (3), I(1)	⊥ (1)
⊥	I(1)	II(3)	—

If Eq. (4.1) has a rational root, Z, then the conditions for triple-wave resonance are fulfilled, i.e., transverse and second-type longitudinal waves can take part in resonance interaction which results in the first-type longitudinal wave. If the root of Eq. (4.1) is not a rational number, this can be achieved by varying the problem's parameters. Similarly, all possible variants of the spatial interaction between elastic waves can be analyzed.

The possibility of realization of the above regimes of wave interaction was confirmed by analyzing all characteristic parameters. Below, interactions will be considered in which two initial waves propagate in orthogonal directions $k^{(j)}$ determined by the eigenvectors $\Lambda^{(j)}$ of the background-stress-state tensor.

The results of numerical calculations for $|k^{(1)}| = |k^{(2)}| = 1$ are listed in Table 1 (for a liquid-saturated porous medium) and in Table 2 (for a gas-saturated porous medium). In this case, I, II, and $\perp$ denote first- and second-type longitudinal waves and transverse waves, respectively. The intersection of lines and columns shows which of the waves results from the resonance interaction between the first (vertically) and second (horizontally) initial waves. The numbers in brackets denote the variants realized. A dash in the Table corresponds to the interaction between two transverse waves when the necessary conditions for the triple-wave resonance are not satisfied (3.1).

The qualitative differences in the wave resonance interactions for saturation of a porous medium with either a liquid or a gas are quite obvious. Thus, the interaction between two first-type longitudinal waves in a liquid-saturated porous medium can give rise to transverse and second-type longitudinal waves, and in a gas-saturated porous medium this can result only in a transverse wave. The interaction between two second-type longitudinal waves in a liquid-saturated porous medium generates a transverse wave, and such intersection in a gas-saturated porous medium gives transverse and first-type longitudinal waves. As follows from Tables 1 and 2, the energy distributions in a system of interacting waves in liquid- and gas-saturated porous media are different in almost all cases. Only in the interaction between transverse and first-type longitudinal waves does three-train resonance occur in a similar way, i.e., new first-type longitudinal wave is generated that "takes" energy from the initial waves. The total system energy remains constant.

Of particular interest are interaction conditions that cause explosive instability and, as a result, the decay of waves. In this case, wave propagation in a gas-saturated medium is less stable than in a medium saturated with incompressible liquid, e.g., water. In a gas-saturated medium the effect of explosive instability can be realized for almost all wave types. This is in fair agreement with the universally accepted concepts on strong wave decay in gas-saturated layers.

Note that the interaction constants depend on the parameters, particularly on the wave vectors, and vary with them (to the point of sign change), so that the type of wave resonance interaction can also change.

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